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Analytical Solution of Partial Integro Differential Equations Using Laplace Differential Transform Method and Comparison with DLT and DET

Yuvraj Pardeshi

Assistant Professor, Faculty of Mathematics, Shri Sant Gadge Baba College of Engineering and Technology, Bhusawal, Maharashtra, India.

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ABSTRACT

Partial Integro Differential Equations (PIDEs) occur naturally in various fields of science and technology. The main purpose of this paper is to study how to solve linear partial integro differential equations with convolution kernel by using the Laplace-Differential Transform Method (LDTM). This method is a simple and reliable technique for solving such equations. The efficiency and reliability of this method is also illustrated with some examples. The result obtained by this method is compared with the result obtained by Double Laplace Transform and Double Elzaki Transform method.

Keywords: *Double Elzaki Transform, Double Laplace Transform, Differential transform method, Laplace Transform, Maclaurin's series, Partial integro differential equation.*

1. Introduction

In the last few years, theory and application of PIDEs play an important role in the various fields of science and technology. It is very important to know various methods to solve such partial integro differential equations. It is found that, the double Laplace transform [1] and Double Elzaki Transform [2] used to solve PIDEs. Using DLT and DET, linear PIDEs convert into an algebraic equation then after factorizing it and applying their inverses an exact solution of the problems was achieved. Recently, Modified Differential transform method [11] and two dimensional DTM [10] have used to solve linear PIDEs.

Laplace Differential Transform Method is the combination of the Laplace Transform method and one-dimensional Differential Transform Method. Earlier, the LDTM has been used to explore the exact solutions of Linear and Nonlinear Klein Gordon equations [5]. Similarly, the same method has used for nonlinear PDEs with boundary conditions [6] and Wave equations [7]. According to the latest research, LDTM has used to solve nonlinear non homogeneous Partial Differential Equations [8].

In this article, first convert linear Partial Integro Differential equation with convolution kernel into Ordinary differential equation, then solved this ODE by Differential transform method and then an exact or series solution is achieved by applying inverse Laplace transform.

2. Preliminaries

2.1. Partial Integro Differential Equation

The general form of PIDE with convolution kernel is,

$$\sum_{i=0}^{m} a_i\left(\frac{\partial^i u}{\partial x^i}\right)+\sum_{i=0}^{n} b_i\left(\frac{\partial^i u}{\partial t^i}\right)+cu(x,t)+\sum_{i=0}^{r} d_i\int_0^t K_i(t-y)\left(\frac{\partial^i u}{\partial x^i}\right)dy+f(x,t)=0 \qquad (1)$$

Where, a_i, b_i, c and d_i are constants or the functions of x alone and $f(x,t)$, $K_i(t-y)$ are known functions.

2.2. Laplace Transform

The Laplace transform of a function $f(t)$ is define by,

$$\bar{f}(s) = L\{f(t)\} = \int_0^\infty e^{-st} f(t)dt \quad ; \quad t \geq 0$$

Where, s is real or complex.

The Laplace transform of first and second order partial derivatives are as follows:

i) $L\left\{\frac{\partial u}{\partial x}\right\} = \bar{u}_x(x,s)$

ii) $L\left\{\frac{\partial^2 u}{\partial x^2}\right\} = \bar{u}_{xx}(x,s)$

iii) $L\left\{\frac{\partial u}{\partial t}\right\} = s\,\bar{u}(x,s) - u(x,0)$

iv) $L\left\{\frac{\partial^2 u}{\partial t^2}\right\} = s^2\,\bar{u}(x,s) - su(x,0) - u_t(x,0)$

Using Laplace transform convolution theorem becomes,

$$L^{-1}\{\bar{g}(s).\bar{f}(s)\} = f(t) * g(t) = \int_0^t f(y)g(t-y)dy$$

2.3. Double Laplace Transform (DLT) [1]

The Double Laplace transform of a function $f(x,t)$ in the positive quadrant of the xt-plane is defined by,

$$\bar{f}(p,s) = L_x L_t\{f(x,t)\} = \int_0^\infty e^{-ps} \int_0^\infty e^{-st} f(x,t)dxdt \quad ; \quad x,t \geq 0$$

Where, p,s are real or complex.

The double Laplace transform of first and second order partial derivatives are as follows:

i) $L_x L_t\left\{\frac{\partial f}{\partial x}\right\} = p\bar{f}(p,s) - \bar{f}(0,s)$

ii) $L_x L_t\left\{\frac{\partial f}{\partial t}\right\} = s\bar{f}(p,s) - \bar{f}(p,0)$

iii) $L_x L_t\left\{\frac{\partial^2 f}{\partial x^2}\right\} = p^2\bar{f}(p,s) - p\bar{f}(0,s) - \bar{f}_x(0,s)$

iv) $L_x L_t\left\{\frac{\partial^2 f}{\partial t^2}\right\} = s^2\bar{f}(p,s) - s\bar{f}(p,0) - \bar{f}_t(p,0)$

2.4. Double Elzaki Transform (DET) [2]

The Double Elzaki transform of a function $f(x,t)$ in the positive quadrant of the xt-plane is defined by,

$$T(p,s) = E_x E_t\{f(x,t)\} = ps\int_0^{\infty}\int_0^{\infty} e^{-\left(\frac{x}{p}+\frac{t}{s}\right)} f(x,t)dxdt \quad ; \quad x,t \geq 0$$

Where, p,s are real or complex.

The Double Elzaki transform of first and second order partial derivatives are as follows:

$$i)\ E_x E_t\left\{\frac{\partial f}{\partial x}\right\} = \frac{1}{p}T(p,s) - pT(0,s)$$

$$ii)\ E_x E_t\left\{\frac{\partial f}{\partial t}\right\} = \frac{1}{s}T(p,s) - sT(p,0)$$

$$iii)\ E_x E_t\left\{\frac{\partial^2 f}{\partial x^2}\right\} = \frac{1}{p^2}T(p,s) - T(0,s) - pT_x(0,s)$$

$$iv)\ E_x E_t\left\{\frac{\partial^2 f}{\partial t^2}\right\} = \frac{1}{s^2}T(p,s) - T(p,0) - sT_t(p,0)$$

2.5. Differential Transform Method (DTM) [3,4]

Let the function $u(x)$ is arbitrary analytic and continuously differentiable function of x defined on $D = [x,0] \subset R$ and $x_0 \in D$ then,

$$U(k) = \frac{1}{k!}\left\{\frac{\partial^k}{\partial x^k}\, u(x)\right\}_{\{x=x_0\}} \quad ; \quad k \geq 0$$

$$\therefore\ u(x) = \sum_{k=0}^{\infty} U(k)(x-x_0)^k \quad ; \quad k \geq 0$$

Note: If $x_0 = 0$ then $\quad \therefore\ u(x) = \sum_{k=0}^{\infty} U(k)x^k \quad ; \quad k \geq 0$

We give the proofs of properties of one dimensional DTM in the following theorem:

Theorem 1: If $U(k)$ is differential transform of $u(x)$ then

$a)$ If $u(x) = x^m$ then $U(k) = \delta(k-m) = \begin{cases} 1 & ; \quad k = m \\ 0 & ; \quad k \neq m \end{cases}$

$b)$ If $u(x) = e^{ax}$ then $U(k) = \dfrac{a^k}{k!}$

$c)$ If $u(x) = sinax$ then $U(k) = \dfrac{a^k}{k!}sin\left(\dfrac{k\pi}{2}\right)$

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d) If $u(x) = cosax$ then $U(k) = \frac{a^k}{k!} cos\left(\frac{k\pi}{2}\right)$

e) If $u(x) = \frac{\partial^m y}{\partial x^m}$ then $U(k) = (k+1)(k+2)\dots(k+m)Y(k+m)$

f) If $u(x) = f(x)g(x)$ then $U(k) = \sum_{m=0}^{k} F(m)G(k-m)$

Proof: a) Let, $u(x) = x^m$

$$\therefore k!\,U(k) = \left\{\frac{\partial^k}{\partial x^k}\, x^m\right\}_{x=0}$$

$$= \{m(m-1)\dots(k-m+1)x^{m-k}\}_{x=0}$$

If $k = m \quad \therefore U(k) = 1$

If $k \neq m \quad \therefore U(k) = 0$

$$\therefore \; U(k) = \delta(k-m) = \begin{cases} 1 & ; \quad k = m \\ 0 & ; \quad k \neq m \end{cases}$$

b) The proof is obvious.

c) Let, $u(x) = sinax$

$$\therefore k!\,U(k) = \left\{\frac{\partial^k}{\partial x^k}\, sinax\right\}_{x=0}$$

$$= \left\{a^k \sin\left(ax + \frac{k\pi}{2}\right)\right\}_{x=0} \qquad \therefore U(k) = \frac{a^k}{k!}\sin\left(\frac{k\pi}{2}\right)$$

Similarly we can prove (d).

e) Let, $u(x) = \frac{\partial^m y}{\partial x^m}$

$$\therefore k!\,U(k) = \left\{\frac{\partial^k}{\partial x^k}\,\frac{\partial^m y}{\partial x^m}\right\}_{x=0}$$

$$= (k+m)!\,Y(k+m) \qquad \therefore U(k) = (k+1).(k+2)\dots(k+m)Y(k+m)$$

f) Let, $u(x) = f(x)g(x)$

$$\therefore k!\,U(k) = \left\{\frac{\partial^k}{\partial x^k}\, f(x)g(x)\right\}_{x=0}$$

$$= \left\{\sum_{m=0}^{k} \frac{\partial^m f}{\partial x^m}.\frac{\partial^{k-m} g}{\partial x^{k-m}}\right\}_{x=0} \qquad \therefore U(k) = \sum_{m=0}^{k} F(m)G(k-m)$$

We summarize the properties of one dimensional DTM in the following table:

Table 1. Properties of one dimensional DTM

Original Function $\boldsymbol{u}(\boldsymbol{x})$	Transformed function $\boldsymbol{U}(\boldsymbol{k})$
α	$\begin{cases}\alpha \;;\; k=0 \\ 0 \;;\; k\neq 0\end{cases}$
x^m	$\begin{cases}1 \;;\; k=m \\ 0 \;;\; k\neq m\end{cases}$
e^{ax}	$\frac{a^k}{k!}$
$sinax$	$\frac{a^k}{k!}\sin\left(\frac{k\pi}{2}\right)$
$cosax$	$\frac{a^k}{k!}\cos\left(\frac{k\pi}{2}\right)$
$\frac{\partial y}{\partial x}$	$(k+1)Y(k+1)$
$\frac{\partial^m y}{\partial x^m}$	$(k+1).(k+2)\dots(k+m)Y(k+m)$
$f(x)g(x)$	$\sum_{m=0}^{k} F(m)G(k-m)$

3. Applications to Linear PIDE

3.1. Double Laplace Transform Method

Applying double Laplace transform to (1),

$$\sum_{i=0}^{m} a_i\left[p^i\bar{u}(p,s)-\sum_{j=0}^{i-1}p^{i-1-j}\,L_t\left(\frac{\partial^j}{\partial x^j}u(0,t)\right)\right]+\sum_{i=0}^{n} b_i\left[s^i\bar{u}(p,s)-\sum_{k=0}^{i-1}s^{i-1-k}\,L_x\left(\frac{\partial^k}{\partial t^k}u(x,0)\right)\right]$$

$$+c\,\bar{u}(p,s)+\sum_{i=0}^{r} d_i\,\bar{k}(s)\left[\sum_{j=0}^{i-1}p^{i-1-j}\,L_t\left(\frac{\partial^j}{\partial x^j}u(0,t)\right)\right]+\bar{f}(p,s)=0$$

This is an algebraic equation in $\bar{u}(p,s)$. After factorizing it and applying inverse DLT, an exact solution of (1) will be achieved.

3.2. Double Elzaki Transform Method

Applying double Elzaki transform to (1),

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$$\sum_{i=0}^{m} a_i \left[\frac{1}{p^i} T(p,s) - \sum_{j=0}^{i-1} p^{2-i+j}\, E_t\left(\frac{\partial^j}{\partial x^j} T(0,t)\right)\right] + \sum_{i=0}^{n} b_i \left[\frac{1}{s^i} T(p,s) - \sum_{k=0}^{i-1} s^{2-i+j}\, E_x\left(\frac{\partial^k}{\partial t^k} T(x,0)\right)\right]$$

$$+\, c\, T(p,s) + \sum_{i=0}^{r} d_i\, \bar{k}(s) \left[\sum_{j=0}^{i-1} p^{2-i+j}\, E_t\left(\frac{\partial^j}{\partial x^j} T(0,t)\right)\right] + F(p,s) = 0$$

This is an algebraic equation in $T(p,s)$. After factorizing it and applying inverse DET, an exact solution of (1) will be achieved.

3.3. Laplace Differential Transform Method

Applying Laplace transform w. r. t t to (1),

$$\sum_{i=0}^{m} a_i\, L\left\{\frac{\partial^i u}{\partial x^i}\right\} + \sum_{i=0}^{n} b_i\, L\left\{\frac{\partial^i u}{\partial t^i}\right\} + c\, L\{u(x,t)\} + \sum_{i=0}^{r} d_i\, L\left\{k(t) * \frac{\partial^i u}{\partial x^i}\right\} + L\{f(x,t)\} = 0$$

$$\therefore \sum_{i=0}^{m} a_i \frac{\partial^i U}{\partial x^i} + \sum_{i=0}^{n} b_i \frac{\partial^i U}{\partial t^i} + c\, U(x,s) + \sum_{i=0}^{r} d_i \left\{K(s) \frac{\partial^i U}{\partial x^i}\right\} + F(x,s) = 0 \qquad (2)$$

Equation (2) is an ODE. This ODE will be solved by one dimensional differential transform method and then by inverse LT, an exact solution of linear PIDE will be achieved.

4. Results and Discussion

Example 1: Consider PIDE

$$u_{tt} = u_x + 2\int_0^t (t-\alpha) u(x,\alpha)\, d\alpha - 2e^x \qquad (3)$$

With initial conditions $u(x,0) = e^x$, $u_t(x,0) = 0$ and boundary condition $u(0,t) = cost$.

Double Laplace Transform [1]: Applying DLT, we get

$$\left(p - \frac{2}{s^2} + s^2\right) U(p,s) = \frac{s}{s^2+1} + \frac{2}{s(p-1)} - \frac{s}{p-1}$$

$$\therefore U(p,s) = \frac{s}{(p-1)(s^2+1)} \qquad (4)$$

Applying inverse DLT,

$$\therefore\ u(x,t) = e^x cost$$

Double Elzaki Transform [2]: Applying DET, we get

$$\left(\frac{1}{p} + 2s^2 - \frac{1}{s^2}\right) T(p,s) = \frac{ps^2}{s^2+1} - \frac{2p^2 s^2}{(1-p)(s^2+1)} - \frac{p^2}{1-p}$$

$$\therefore T(p,s) = \frac{p^2 s^2}{(p-1)(s^2+1)} \qquad (5)$$

Applying inverse DET,

$$\therefore\ u(x,t) = e^{x} cost$$

Laplace-Differential Transform method:

Applying LT with respect to t on (3) and using $u(x,0) = e^{x},\ u_t(x,0) = 0$

$$\therefore \frac{dU(x,s)}{dx} + \left(\frac{2}{s^2} - s^2\right) U(x,s) = e^{x}\left(\frac{2}{s} - s\right) \tag{6}$$

Now applying DTM

$$\bar{U}(k+1,s) = \frac{1}{(k+1)}\left\{\left(\frac{s^4-2}{s^2}\right)\bar{U}(k,s) + \frac{1}{k!}\left(\frac{2}{s} - s\right)\right\} \tag{7}$$

Now applying the Laplace transform to boundary condition

$$U(0,s) = \frac{s}{s^2+1} \quad \therefore \quad \bar{U}(0,s) = \frac{s}{s^2+1}$$

Put $k = 0, 1, 2, 3, ...$ in equation (7),

If $k = 0$; $\bar{U}(1,s) = \frac{s}{s^2+1}$ If $k = 1$; $\bar{U}(2,s) = \frac{1}{2!}\left(\frac{s}{s^2+1}\right)$

If $k = 2$; $\bar{U}(3,s) = \frac{1}{3!}\left(\frac{s}{s^2+1}\right)$ If $k = 3$; $\bar{U}(4,s) = \frac{1}{4!}\left(\frac{s}{s^2+1}\right)$ and so on ...

Now, $U(x,s) = \sum_{k=0}^{\infty} \bar{U}(k,s)(x)^k = \frac{s}{s^2+1}\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\right)$

Now applying Inverse LT,

$\therefore u(x,t) = cost\ e^{x}$ (Also shown in graphical presentation: see **Fig. 1**)

Example 2: Consider PIDE

$$u_t - u_{xx} + u + \int_0^t e^{(t-\alpha)} u(x,\alpha) d\alpha = (x^2+1)e^{t} - 2 \tag{8}$$

With initial conditions $u(x,0) = x^2$, $u_t(x,0) = 1$ and boundary conditions $u(0,t) = t,\ u_x(0,t) = 0$

Double Laplace Transform [1]: Applying DLT, we get

$$\left(s - p^2 + 1 - \frac{1}{s-1}\right) U(p,s) = \frac{2+p^2}{p^3(s-1)} - \frac{2}{ps} + \frac{2}{p^3} - \frac{p}{s^2}$$

$$\therefore U(p,s) = \frac{2}{p^3 s} + \frac{1}{ps^2} \tag{9}$$

Applying inverse DLT,

$$\therefore\ u(x,t) = x^2 + t$$

Double Elzaki Transform [2]: Applying DET, we get

$$\left(\frac{1}{s}-\frac{1}{p^2}+1+\frac{s}{1-s}\right)T(p,s)= \quad (2p^2+1)\left(\frac{s^2p^2}{1-s}\right)-2p^2s^2+2sp^4-s^3$$

$$\therefore T(p,s)=2p^4s^2+p^2s^3 \qquad (10)$$

Applying inverse DET,

$$\therefore\ u(x,t)=x^2+t$$

Laplace-Differential Transform method:

Applying LT with respect to t on (8) and using $u(x,0)=x^2$, $u_t(x,0)=1$

$$\therefore\ \frac{d^2U(x,s)}{dx^2}-\left(\frac{s^2}{s-1}\right)U(x,s)=\frac{2}{s}-\frac{(x^2+1)}{s-1}-x^2 \qquad (11)$$

Applying DTM,

$$\overline{U}(k+2,s)\ =\frac{1}{(k+2)(k+1)}\left\{\left(\frac{s^2}{s-1}\right)\overline{U}(k,s)-\delta(k-2)\left(\frac{s}{s-1}\right)-\frac{1}{s-1}\delta(k)+\frac{2}{s}\delta(k)\right\} \qquad (12)$$

Now applying the LT to boundary conditions we get,

$$\therefore\ \ U(0,s)=\frac{1}{s^2} \qquad and \qquad \frac{dU(0,s)}{dx}=0$$

Applying DTM, $\quad \therefore\ \ \overline{U}(0,s)=\frac{1}{s^2} \qquad and \quad \overline{U}(1,s)=0$

Put $k=0,1,2,3,\ldots$ in equation (12),

If $k=0$ $\quad \therefore \overline{U}(2,s)=\frac{1}{s}$; If $k=1$ $\quad \therefore \overline{U}(3,s)=0$

If $k=2$ $\quad \therefore \overline{U}(4,s)=0$; If $k=3$ $\quad \therefore \overline{U}(5,s)=0$ and so on ...

Now, $\quad U(x,s)=\sum_{k=0}^{\infty}\overline{U}(k,s)(x)^k=\ \frac{1}{s^2}+\frac{1}{s}x^2$

$\therefore$ Applying inverse LT

$\therefore u(x,t)=x^2+t$ (Also shown in graphical presentation: see **Fig. 2**)

Example 3: Consider PIDE

$$u_t+u_{ttt}-\int_0^t sinh(t-\alpha)\,u_{xxx}(x,\alpha)d\alpha=0 \qquad (13)$$

With $u(x,0)=0$, $u_t(x,0)=x$, $u_{tt}(x,0)=0$ and $u(0,t)=0$, $u_x(0,t)=sint$, $u_{xx}(0,t)=0$.

Double Laplace Transform [1]: Applying DLT, we get

$$\left(s+s^3-\frac{p^3}{s^2-1}\right)U(p,s)=\frac{s}{p^2}-\frac{p}{(s^2-1)(s^2+1)}$$

$$\therefore U(p,s) = \frac{1}{p^2(s^2+1)} \tag{14}$$

Applying inverse DLT,

$$\therefore\ u(x,t) = xsint$$

Double Elzaki Transform [2]: Applying DET, we get

$$\left(\frac{1}{s} + \frac{1}{s^3} - \frac{1}{1-p^2}\right) T(p,s) = \ p^3 - \frac{p^3 s^3}{(1-p^2)(s^2+1)}$$

$$\therefore T(p,s) = \frac{p^3 s^3}{s^2+1} \tag{15}$$

Applying inverse DET,

$$\therefore\ u(x,t) = xsint$$

Laplace-Differential Transform method:

Applying LT with respect to t on (7) and using $u(x,0) = 0,\ u_t(x,0) = x,\ u_{tt}(x,0) = 0$

$$\therefore\ \frac{d^3}{dx^3} U(x,s) - (s^2-1)(s+s^3)U(x,s) + sx(s^2-1) = 0 \tag{16}$$

Now applying DTM

$$\bar{U}(k+3,s) = \frac{1}{(k+1)(k+2)(k+3)} \{(s^5 - s)\bar{U}(k,s) + (s - s^3)\delta(k-1)\} \tag{17}$$

Now applying the LT on boundary conditions we get,

$$U(0,s) = 0, \qquad \frac{dU(0,s)}{dx} = \frac{1}{s^2+1} \quad and \quad \frac{d^2U(0,s)}{dx^2} = 0$$

Applying DTM, $\bar{U}(0,s) = 0,\quad \bar{U}(1,s) = \frac{1}{s^2+1}\quad and\quad \bar{U}(2,s) = 0$

Put $k = 0,1,2$ ... in equation (17) we get,

If $k = 0$ $\therefore \bar{U}(3,s) = 0$ If $k = 1$ $\therefore \bar{U}(4,s) = 0$

If $k = 2$ $\therefore \bar{U}(5,s) = 0$ If $k = 3$ $\therefore \bar{U}(6,s) = 0$ and so on ....

$$Now, \quad U(x,s) = \sum_{k=0}^{\infty} \bar{U}(x,s)(x)^k = \frac{1}{s^2+1}x$$

Applying inverse Laplace Transform

$\therefore u(x,t) = xsint$ (Also shown in graphical presentation: see **Fig.3**)

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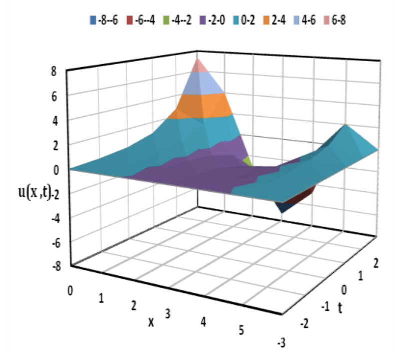

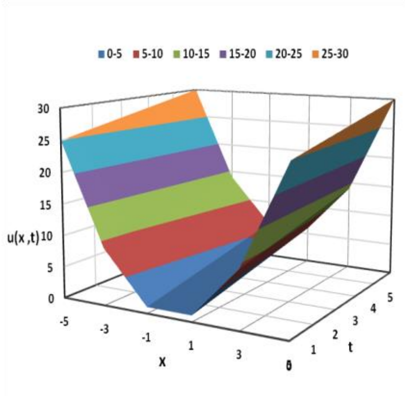

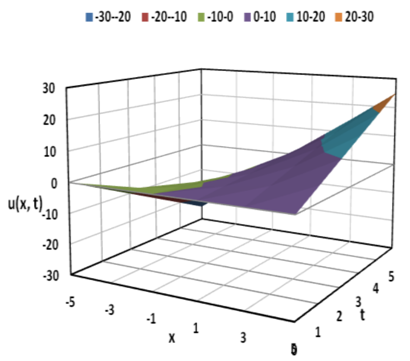

Fig.1. Solution of $u(x,t) = e^x cost$ **Fig.2.** Solution of $u(x,t) = x^2 + t$ **Fig.3.** Solution of $u(x,t) = xsint$

In this section, we have solved linear PIDEs with a convolution kernel of different order by using DLT, DET and LDTM. The graphical solution also been presented. It is observed that the DLT method reduces the linear PIDE with convolution kernel into algebraic equations (4), (9), and (14). Similarly, the DET method also reduces linear PIDEs into algebraic equations (5), (10) and (15). These algebraic equations take a lot of computational effort and time for their factorization and to achieve an exact solution of the problem. It has also been observed that the same solutions achieved by using LDTM method too. In the LDTM method, the Laplace transform reduces linear PIDEs into ordinary differential equations which are solved by the Differential Transform Method then the series solution of linear PIDEs is obtained by applying the inverse Laplace transform. In some cases, an exact solution can be achieved.

5. Conclusion

In this paper, the Laplace Differential Transform Method was successfully applied on linear Partial Integro Differential Equations with a convolution kernel. It is observed that, the solutions obtained by the LDTM method are the same as the DLT and DET methods. The LDTM gives a series solution and in some cases, the exact solution can be achieved. We firmly ascertained that LDTM takes less computational effort and time than others. It is concluded that LDTM is an alternate method to solve such kind of PIDEs. This method is a very simple, powerful and reliable for solving such problems. We hope some other types of PIDE can be used in various fields of modeling real life phenomena.

Declarations

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Competing Interests Statement

The author declares no competing financial, professional and personal interests.

Consent for publication

Author declares that he/she consented for the publication of this research work.

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